ISI Type-A Mock Test

Circle the correct option. Correct Answer = 4 marks, Leave Blank = 1, Wrong Answer = 0

Name:
Date:

1. The sequence defined by $x_{n+1} = \sqrt{ax_n^2 + b}$, where $x_1 = c$ converges to a real number if;
A. $0 < a < 1$ and $b < 0$ B. $0 < a < 1$ and $b > 0$ C. $a > 1$ and $b < 0$ D. $a > 1$ and $b > 0$

2. The number of intersection point between the curves $r = \sin(\theta/2)$ and $r = \cos(\theta/2)$, where $r \geq 0, \theta \in [0, 2\pi]$ in polar coordinates is; A. None. B. 1 C. More than 1, but finitely many. D. Infinitely many.

3. The limit; $\lim_{x\to\infty} x(\exp(1/x) - 1)$ is; A. 0. B. 1. C. e. D. Does not exist.

4. A rectangle is inscribed into a sector of circle of radius 1, making a central angle of $\pi/6$. The maximum possible area of the rectangle is; A. $2\sqrt{3}$. B. $1/2$. C. $(2 - \sqrt{3})/2$. D. $(2 + \sqrt{3})/2$.

5. It is given that; $\sum_{n=1}^{\infty} a_n$ converges. Which of the following is true? A. $\sum_{n=1}^{\infty} a_n^2$ converges. B. $\sum_{n=1}^{\infty} \sqrt{a_n}$ converges. C. $\sum_{n=1}^{\infty} a_n \sin(a_n)$ diverges. D. $\sum_{n=1}^{\infty} a_n/\log(a_n)$ diverges.

6. Consider the polynomial $P(x) = (1+x)^{2019} + x(1+x)^{2018} + x^2(1+x)^{2017} + \ldots x^{2019}$. Then, the coefficient of x^{50} is; A. 2020^5 B. $\binom{2019}{5}5!$ C. $\binom{2020}{5}5!$ D. $\binom{2020}{5} + \binom{2019}{5} + \binom{2018}{5} + \binom{2017}{5} + \binom{2016}{5}$

7. The number of m such that 2^m divides $(3^m - 1)$ is; A. None. B. 1. C. 2. D. 3.

8. The number of real roots of the equation; $3^x - 2x^2 = 1$ is; [**Hint:** $1 < \ln 3 < 1.1$] A. None. B. 1. C. 3. D. More than 3.

9. In $\triangle ABC$, the angle $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius, then $2(r + R) =$?
A. $(a + b)$ B. $(a + c)$ C. $(b + c)$ D. $(a + b + c)$

10. The number of integral values of k for which $7\cos x + 5\sin x = 2k + 1$ has a solution of x is; A. 4 B. 8 C. 10 D. 12

11. If a complex number z be such that $|z^2 - 1| = |z|^2 + 1$, then the locus of z is; A. A circle B. An ellipse C. Real axis D. Imaginary axis

12. If z is a 2019-th root of unity, then the limit $\lim_{n\to\infty}(z + z^2 + \ldots z^{2017} - z^{2018})^n/3^n$ is; A. Does not exist B. 0 C. 1 D. (-1)

13. The series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots$ is equal to; A. $(e^2 - 1)/2$ B. $(e - 1)^2/2e$ C. $(e^2 - 1)/2e$ D. $(e^2 - 2)/e$

14. If x, y, z are the p, q and r-th term of an arithmetic progression respectively, then the value of $x^{(y-z)}y^{(z-x)}z^{(x-y)}$ is; A. xyz B. 0 C. $(p + q + r)$ D. None of these.

15. A rectangle is constructed of length $(2m - 1)$ and breadth $(2n - 1)$ units respectively. The rectangle is divided into small unit rectangular grids by drawing parallel lines to its sides. The total number of rectangles having odd side lengths which can be inscribed within the large rectangle is; A. $m^2 - n^2$ B. $mn(m + 1)(n + 1)$ C. $4^{(m+n-2)}$ D. m^2n^2

16. The maximum number of regions in which 10 circles can divide a plane is; A. 1024 B. 512 C. 92 D. None of these.

17. A point (x, y) in the plane satisfying the equation $x^2 + 2x\sin(xy) + 1 = 0$ lie on; A. A pair of straight lines B. A family of hyperbolas C. A parabola D. An ellipse

18. Two ants are both standing on the origin of the number line, and both have to go right or left by one unit in each step,, irrespective of the other. Then in how many ways can they travel along the number line such that after n steps, they are standing at the same point? A. 2^n B. $\binom{n}{2}$ C. $\binom{2n}{n}$ D. 4^n

19. What is the sum of cubes of divisors of 72? A. 373249 B. 433729 C. 441289 D. 442845

20. What is the remainder of dividing $3333^{4444} + 4444^{3333}$ by 7? (Can you think for more problems like this?)
A. 0 B. 1 C. 2 D. 5